ACT 2020, MIDTERM #1
ECONOMIC AND FINANCIAL APPLICATIONS
FEBRUARY 11, 2009
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Answer Key

You have 70 minutes to complete this exam. When the invigilator instructs you to stop writing you must do so immediately. If you do not abide by this instruction you will be penalised. All invigilators have full authority to disqualify your paper if, in their judgement, you are found to have violated the code of academic honesty.

Each question is worth 10 points. Provide sufficient reasoning to back up your answer but do not write more than necessary.

This exam consists of 8 questions. Answer each question on a separate page of the exam book. Write your name and student number on each exam book that you use to answer the questions. Good luck!

Question 1. Bill and Jim enter into a binding contract involving the exchange of an asset in nine months time. The current market price of the asset is \$100. The continuously compounded interest rate is $r = 0.10$. The contract calls for Jim to sell the asset to Bill at that time for a price of 105. Furthermore, Jim and Bill agree that the fair value of the contract at initiation is to be zero and that, if necessary, an amount will be fixed at contract initiation that one will pay to the other in nine months time so that this condition is met. If the price of the asset in nine months time is \$104 what is Jim's total profit or loss at the time the asset is sold?

Question 2. Assume that you open a 100 share short position in Jiffy Inc. common stock at the bid-ask price of \$32.00 - \$32.50. When you close your position after exactly one-year, the bid-ask prices are \$29.50 - \$30.25. Jiffy Inc. common stock paid a dividend of \$1.15 exactly 3 months after you opened your short position. If you pay a commission rate of 1.5%, and the continuously compounded interest rate is 3%, calculate your profit or loss on this trade at the time the position is closed?

Question 3. John has just purchased a home for $100,000. The insurance policy will cover any losses due to fire subject to a deductible of $10,000. The one-year premium on the insurance policy is $1,000 which is due at the start of the policy year.

(1) [5 points] Let M denote the market value of John's home at the end of the policy year.[1] Draw a chart of the net value of John's home and insurance policy versus M at the end of the policy year. Assume the continuously compounded interest rate is $r = 0.10$.

(2) [3 points] Explain in what sense there is a put option implicit in this transaction, possibly drawing an appropriate chart.

(3) [2 points] At what value of M would John have the same wealth with and without the above insurance policy? If there is no such value of M explain why.

Question 4. Barkley Corporation (BC) has decided to issue a mandatory convertible bond. The bond matures at time T, pays a 5% annual coupon on a $1,000 notional face amount, and the share price at that time is denoted S_T. The mandatory convertible bond pays the bondholder 25 shares at bond maturity if the share price is below $40, it pays the bondholder $(25 \cdot 40)/S_T$ BC shares at bond maturity if the share price is between $40 and $60 and it pays the bondholder 16 and 2/3 shares at bond maturity if the share price is above $60.

(1) [3 points] Draw a chart of the payoff at bond maturity to the owner of the Barkley Corporation mandatory convertible bond. (As was done in class, your chart should not include the interest component of the bond.)

(2) [2 points] If it turns out that $S_T = 39$, compute the dollar amount of the payoff at bond maturity to the owner of the Barkley Corporation mandatory convertible bond.

(3) [5 points] The President of Barkley Corporation, is also interested in the possibility of issuing a convertible bond. This bond would pay a 2% annual coupon on a $1,000 notional face amount, and would permit the bondholder to receive the face amount of $1,000 at maturity or 25 shares in Barkley Corp., to be determined at the bondholder's discretion. Assume that $T = 2$ for this bond and the mandatory convertible bond and that the continuously compounded force of interest is $r = 0.04$. For what value of the share price at bond maturity, *i.e.* S_2, would a person that bought the mandatory convertible bond have the same payoff as a person that bought the convertible bond? If there is no such value for S_2 explain why.

[1] If there is a devastating fire the market value of John's home could be close to zero. If there is no fire and the real estate market soars then the market value of John's home could be more than John's purchase price of $100,000.

Question 5. The Federated Bank of Canada is offering a structured product that guarantees a continuously compounded return of 3% on the investor's capital at the end of 5 years. The continuously compounded interest rate is $r = 0.08$. The index underlying the structured product is currently at 1200 (*i.e.* $S_0 = 1200$). The current market price of one at-the-money European call on the underlying index expiring in 5 years is \$450.

(1) [4 points] Assuming that the Federated Bank of Canada charges the investor a front-end load of 1% for this contract (*i.e.* the haircut is 1%), compute the participation rate for the structured product.

(2) [3 points] Now consider exactly the same contract and market data except that the investor is now guaranteed to receive 90% of their capital at the end of 5 years. Assuming that the Federated Bank of Canada charges the investor a front-end load of 1% for this contract (*i.e.* the haircut is 1%), compute the participation rate for the structured product.

(3) [3 points] At what effective rate of return for the index over the five-year period will an investor receive the same rate of return on either of the contracts defined in **(1)** and **(2)** above? If no such rate of return exists, explain why.

Question 6. Barkley & Ruff Cereal Company sells "Sugar Corns" for \$2.50 per box. The company will need to buy 20,000 bushels of corn in 6 months to produce 40,000 boxes of cereal. Non-corn costs total \$60,000. What is the company's profit if they purchase call options at \$0.12 per bushel with a strike price of \$1.60? Assume the 6-month interest rate is 4.0% and the spot price in 6 months is \$1.65 per bushel.

Question 7.

Happy Jalapenos, LLC has an exclusive contract to supply jalapeno peppers to the organizers of the annual jalapeno eating contest. The contract states that the contest organizers will take delivery of 10,000 jalapenos in one year at the market price. It will cost Happy Jalapenos 1,000 to provide 10,000 jalapenos and today's market price is 0.12 for one jalapeno. The continuously compounded risk-free interest rate is 6%.

Happy Jalapenos has decided to hedge as follows (both options are one-year, European):

Buy 10,000 0.12-strike put options for 84.30 and sell 10,000 0.14-stike call options for 74.80.

Happy Jalapenos believes the market price in one year will be somewhere between 0.10 and 0.15 per pepper. Which interval represents the range of possible profit one year from now for Happy Jalapenos?

A. –200 to 100
B. –110 to 190
C. –100 to 200
D. 190 to 390
E. 200 to 400

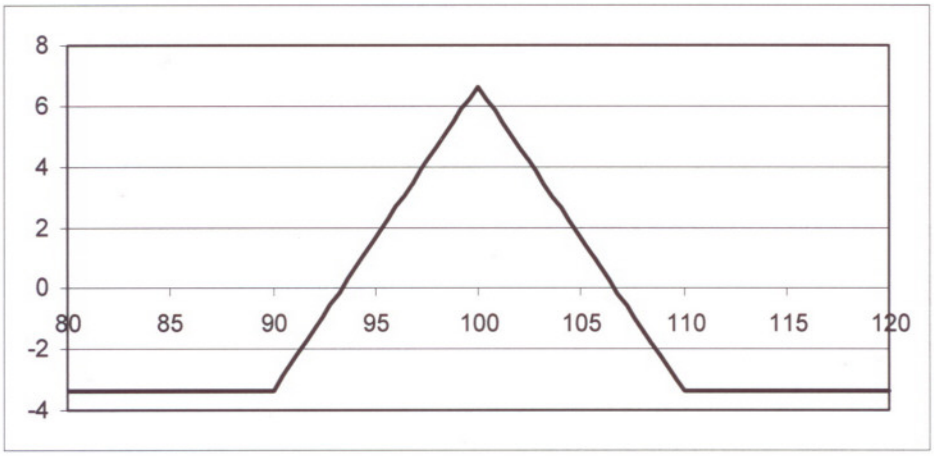
Question 8.

You are given the following information:

- The current price to buy one share of ABC stock is 100
- The stock does not pay dividends
- The risk-free rate, compounded continuously, is 5%
- European options on one share of ABC stock expiring in one year have the following prices:

Strike Price	Call option price	Put option price
90	14.63	0.24
100	6.80	1.93
110	2.17	6.81

A butterfly spread on this stock has the following profit diagram.



Which of the following will NOT produce this profit diagram?

A. Buy a 90 put, buy a 110 put, sell two 100 puts
B. Buy a 90 call, buy a 110 call, sell two 100 calls
C. Buy a 90 put, sell a 100 put, sell a 100 call, buy a 110 call
D. Buy one share of the stock, buy a 90 call, buy a 110 put, sell two 100 puts
E. Buy one share of the stock, buy a 90 put, buy a 110 call, sell two 100 calls.

Solutions:

Question 1. The easiest way to approach this problem is to note that after the payment is made rendering this a zero cost contract that Jim and Bill have agreed to a forward contract. Jim has agreed to deliver the asset and he is therefore going to have to purchase the asset at its market price on the delivery date in order to fulfil his committment to deliver the asset to Bill. Therefore, Jim's profit or loss depends on the forward price less the market price of the asset to be delivered on the delivery date.

The forward price is $100 \cdot e^{0.10(0.75)} = 107.79$. Jim's profit is $107.79 - 104.00 = 3.79$.

$$\text{Answer} = 3.79$$

One can also look at the problem in the following way. Let the value of the asset at time 0.75 be denoted $S_{0.75}$. Jim will receive the cash flow $105 - S_{0.75}$ at time 0.75. The value of this cash flow today is

$$105 \cdot e^{-0.10(0.75)} - S_0 = 105 \cdot e^{-0.075} - 100 = -2.59$$

Therefore, Bill must pay Jim $2.59 \cdot e^{0.10(0.75)} = 2.79$ at time 0.75 for this to be a zero value contract. Jim's profit is then the sum of this payment and his net cost to deliver the asset at time 0.75. Therefore, Jim's profit is

$$2.79 + (105 - 104) = 3.79$$

□

Question 2. The proceeds from the short sale at time 0 are

$$100(32.00)(1 - 0.015) = 3152.00$$

The accumulated value of the short sale proceeds after 1 year is

$$3152 \cdot e^{0.03(1)} = 3247.99$$

The accumulated value at time 1 of the dividend payment that you must make at time 0.25 because you are short the shares is

$$1.15(100) \cdot e^{0.03(0.75)} = 117.62$$

The cost to cover your position (*i.e.* repurchase the shares you are short) at time 1 is

$$100(30.25)(1 + 0.015) = 3070.38$$

Your profit or loss is equal to the "accumulated value of the short sale proceeds" less the "accumulated value of required dividend payments made" less the "cost to cover your position". Therefore, your profit is

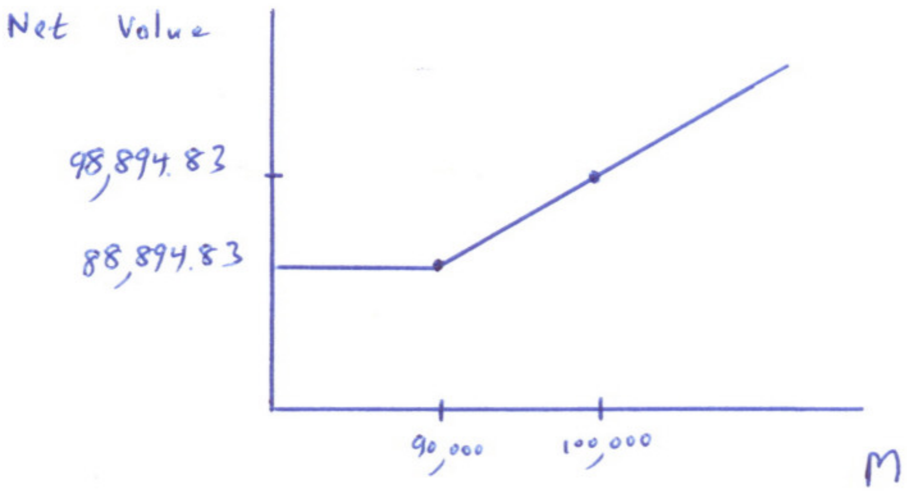
$$3247.99 - 117.62 - 3070.38 = 59.99$$

$$\text{Answer} = 59.99$$

□

Question 3:

(1) $1{,}000\, e^{.10(1)} = 1{,}105.17$



The algebraic expression for the graph/chart is:

$$M + (90{,}000 - M)_+ - 1{,}000\, e^{.10(1)}$$

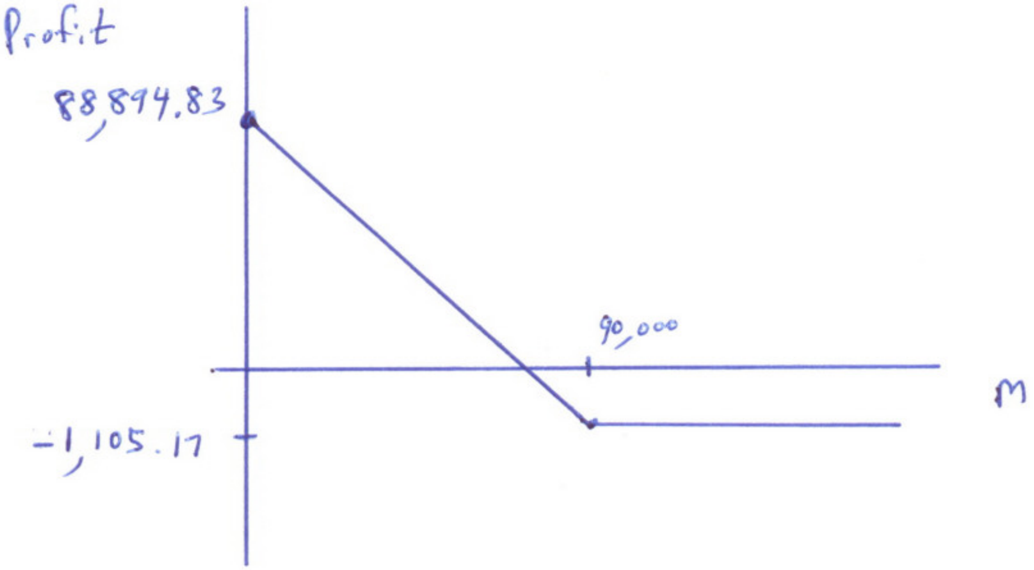
$$= MAX(90{,}000, M) - 1{,}105.17 .$$

(2) The payoff from the insurance policy is a put option on the market value of the house with a strike price of 90,000.

∴ the insurance policy is a put option on the market value of the home with profit at the end of the policy year equal to:

$$(90{,}000 - M)_+ - 1{,}105.17 .$$

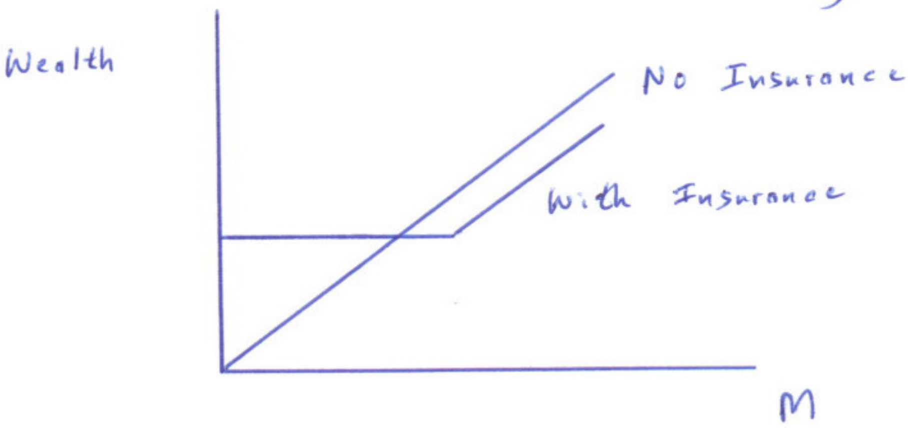
A plot of the put profit is:



Some discussion along these lines is acceptable.

(3) If John does not insure he is always better off than he would be with insurance providing there is no significant loss in market value.

The wealth profiles with and without insurance are shown in the following chart:



We are solving the algebraic equation

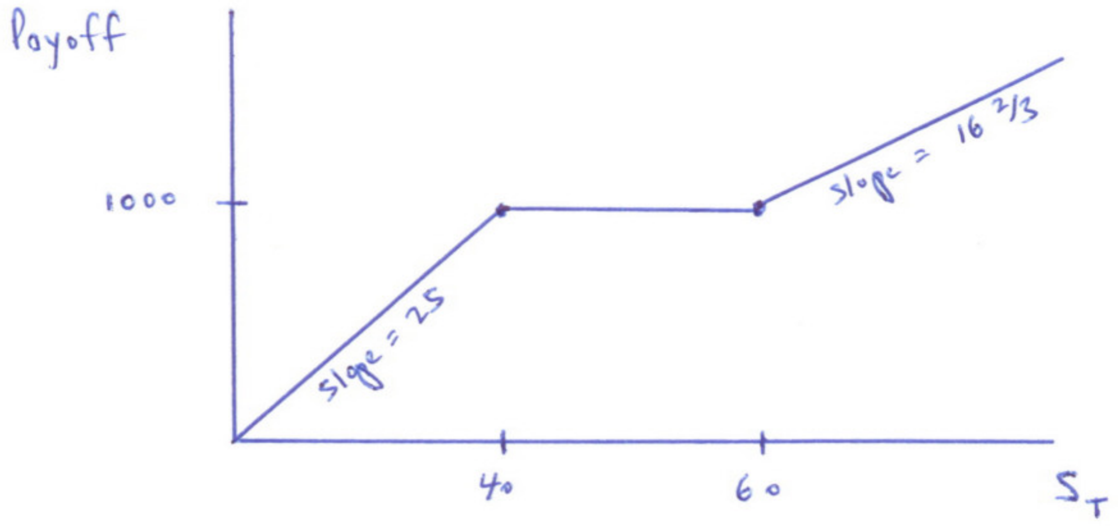
$$M = \text{Max}(90{,}000, M) - 1{,}105.17 .$$

$$\therefore \quad M = 90{,}000 - 1{,}105.17 = 88{,}894.83 .$$

Answer = 88,894.83 . [You are indifferent when the loss equals your deductible plus premium with interest.]

Question 4:

(1) Payoff = # Shares x Share Price



(2) From the picture in (1), the payoff is $25(39) = 975$.

One could also write out the payoff algebraically as:

$$\text{Payoff} = 25 S_T - 25(S_T - 40)_+ + 16\tfrac{2}{3}(S_T - 60)_+$$

but this is not necessary here.

(3) The cash flows from the convertible bond are:

0	1	2
	20	$20 + 1000 + 25(S_2 - 40)_+$

As we discussed in class, the redemption feature has a cash flow equal to:

$$\text{Max}(1000, 25 S_2) = 1000 + (25 S_2 - 1000)_+ = 1000 + 25(S_2 - 40)_+ .$$

The cash flows from the mandatory convertible are:

0	1	2
	50	$50 + \varphi(S_2)$

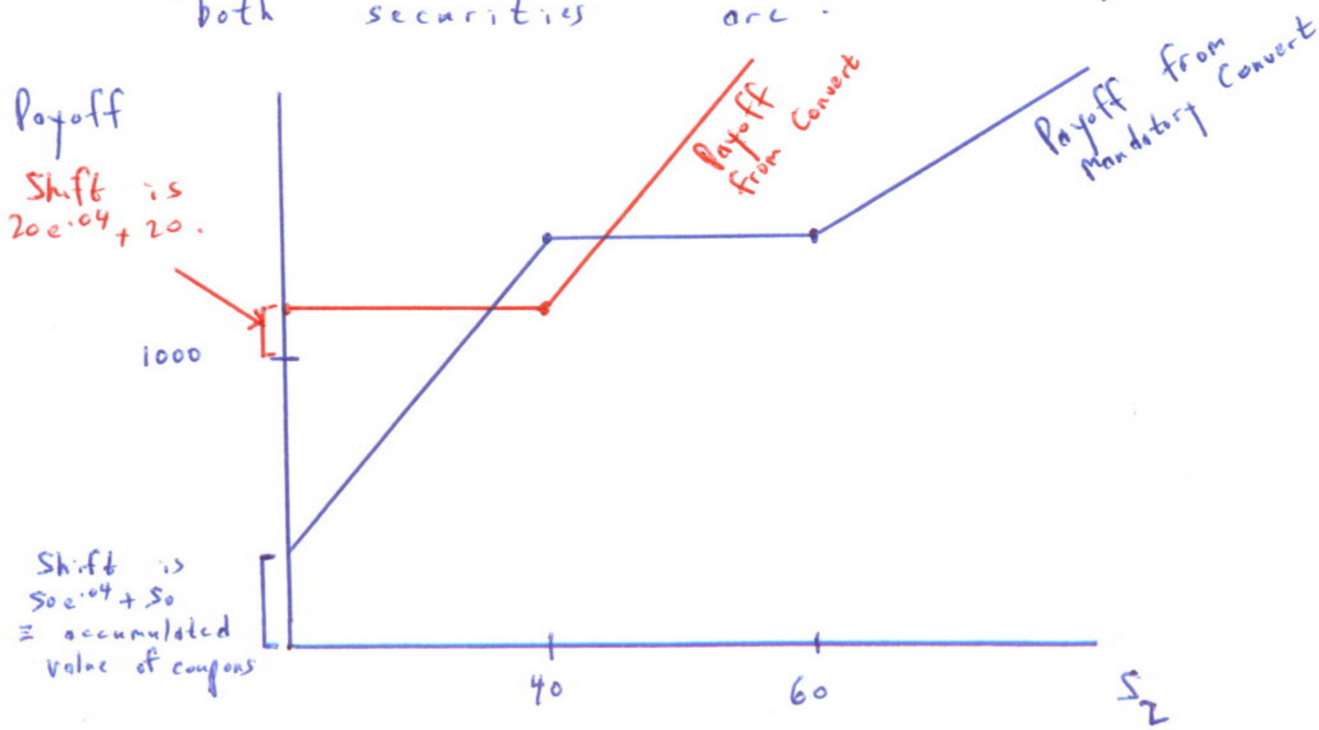
where $\varphi(S_2) = 25S_2 - 25(S_2-40)_+ + 16\frac{2}{3}(S_2-60)_+$

OR φ is as shown on the chart from (1).

$\therefore$ we need to find S_2 so that:

$$20e^{.04} + 20 + 1000 + 25(S_2-40)_+$$
$$= 50e^{.04} + 50 + \varphi(S_2).$$

While this can be solved algebraically, it is useful to note that the general relationship of the payoffs from both securities are:



The key insight is that we expect the two curves to cross in the range $40 < S_2 < 60$. $\therefore$ we are solving:

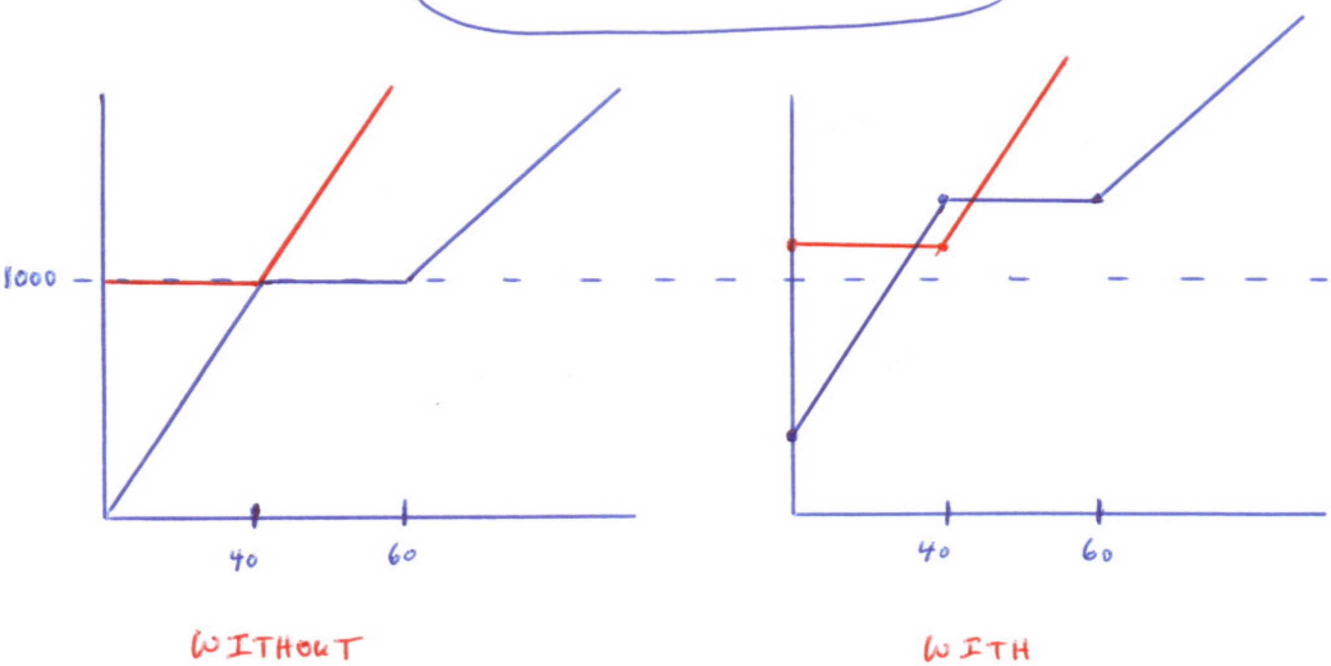
$$20e^{.04} + 20 + 1000 + 25(S_2 - 40)$$

$$= 50e^{.04} + 50 + 1000$$

$$\Rightarrow \quad 25(S_2 - 40) = 61.2243$$

$$\Rightarrow \quad S_2 = 42.45$$

Answer = 42.45



WITHOUT COUPONS

WITH COUPONS

This pair of pictures is all that is needed to quickly reason the solution range for S_2.

Question 5:

(1) I_0 = initial investment

$I_0 e^{.03(5)}$ = guaranteed payoff

$I_0 e^{.03(5)} e^{-.08(5)}$ = amount needed to fund guarantee

$I_0(.01)$ = commission to dealer/bank

$I_0 - I_0 e^{.03(5)} e^{-.08(5)} - I_0(.01)$ = residual amount for investment in at the money call options

Participation Rate = Multiple of Upside Index Return Investor Receives

Investor Payoff Linked to Index = # Call Options Purchased × Call Option Payoff

$$= \frac{\text{Amount Invested in Calls}}{\text{Price of Call}} \times \text{Call Option Payoff}$$

$$= \frac{I_0(1 - e^{.03(5)} e^{-.08(5)} - .01)}{450} \times (S_5 - 1200)_+$$

$$= I_0(1 - e^{.03(5)} e^{-.08(5)} - .01) \frac{1200}{450} \left(\frac{S_5}{S_0} - 1 \right)_+$$

$$1 + \text{Investor Return} = \frac{I_0 e^{.03(5)} + I_0(1 - e^{.03(5)} e^{-.08(5)} - .01) \frac{1200}{450} \left(\frac{S_5}{S_0} - 1 \right)_+}{I_0}$$

$$R = e^{.03(5)} - 1 + \underbrace{(1 - e^{.03(5)} e^{-.08(5)} - .01) \frac{1200}{450}}_{\text{Participation Rate}} (R^{\text{Index}})_+$$

Participation Rate = .5632

Answer = 56.32%

Also, $R = .1618 + .5632\,(R^{\text{Index}})_+$.

<u>Note:</u> A direct calculation of:

$$\text{Participation Rate} = \left(1 - e^{.03(5)}e^{-.08(5)} - .01\right)\frac{1200}{450} = .5632$$

is acceptable.

(2) $$\text{Payoff} = I_0(.90) + \frac{I_0\left(1 - .90e^{-.08(5)} - .01\right)}{450}(S_5 - 1200)_+$$

$$R = -.10 + \left(1 - .90e^{-.08(5)} - .01\right)\frac{1200}{450}\left(R^{\text{Index}}\right)_+$$

$$= -.10 + 1.0312\left(R^{\text{Index}}\right)_+$$

$$\text{Participation Rate} = 1.0312$$

Answer = 103.12 %

A direct calculation of

$$\left(1 - .90e^{-.08(5)} - .01\right)\frac{1200}{450}$$

is acceptable.

(3) From (1) and (2) we have the return equations:

$$R = .1618 + .5632 \, (R^{Index})_+$$

$$R = -.10 + 1.0312 \, (R^{Index})_+$$

Solve $.1618 + .5632 \, z$

$$= -.10 + 1.0312 \, z$$

$\Rightarrow \quad z = .5594$

$$R^{Index} = .5594$$

OR 55.94%

Answer $= 55.94\%$

General Formulation:

T: investment horizon

I_0: initial investment

G: multiple of initial investment that is guaranteed

r: continuously compounded rate of interest

h: haircut/commission/front-end load as % of initial investment

C: price of at-the-money call option on the underlying investment index

S_0: initial index level

S_T: final index level (random)

Examples on G:

90% of initial investment return guarantee: $G = 0.90$

0% return (i.e. 100% return of capital): $G = 1.00$

2% guaranteed continuously compounded return: $G = e^{.02(T)}$

Investment in Bonds to Fund Guarantee: $I_0 G e^{-rT}$

Commission: $I_0 h$

Amount Available to Purchase At-the-Money Call Options: $I_0 - I_0 G e^{-rT} - I_0 h$

$$\text{Investor Payoff} = I_0 G + \frac{(I_0 - I_0 G e^{-rT} - I_0 h)}{C}(S_T - S_0)_+$$

$$\text{Effective Return on Index on } [0,T] = \frac{S_T}{S_0} - 1 \equiv R^{Index}_{[0,T]}$$

$$\text{Investor Effective Return on } [0,T] = \frac{\text{Payoff}}{I_0} - 1$$

$$= (G-1) + (1 - G e^{-rT} - h)\frac{S_0}{C}\left(R^{Index}_{[0,T]}\right)_+$$

$$R_{[0,T]} = (G-1) + (1 - G e^{-rT} - h)\frac{S_0}{C}\left(R^{Index}_{[0,T]}\right)_+$$

$$\text{Participation Ratio} = (1 - G e^{-rT} - h)\frac{S_0}{C}$$

Question 6:

The company's total cost depends on a fixed amount of 60,000 plus 20,000 x (cost of a bushel of corn).

By purchasing call options, the company guarantees they do not pay more than \$1.60 per bushel of corn.

$$\text{Profit} = 40{,}000(2.50) - [60{,}000 + 20{,}000(1.60) + \underbrace{20{,}000(.12)(1.04)}_{\text{Cost of Call Options with Interest}}]$$

Cost of Corn at Guaranteed Price Since Spot Price is Above Strike

$$= 100{,}000 - 94{,}496 = 5504$$

Answer = \$5504 profit

Formal Approach:

Profit = Revenue − Costs

Revenue = Price Per Unit x (# Units Sold)

$= 2.50 \times 40{,}000 = 100{,}000$

$S_{1/2}$ = spot price per bushel of corn in 6 months time

Unhedged Cost = 60,000 + 20,000 $S_{1/2}$

Hedged Cost = Unhedged Cost − Net Cash Flow From Hedge

Net Cash Flow from Hedge = Payoff from Hedge − Cost of Hedge Accounting for Interest

Profit with Hedge = Revenue − Hedged Cost

$$= 100{,}000 - \left[60{,}000 + 20{,}000\, S_{1/2} - \left[20{,}000(S_{1/2} - 1.60) + - 20{,}000(.12)(1.04) \right] \right]$$

For $S_{1/2} = 1.65$,

$$\text{Hedged Profit} = 100{,}000 - \left[60{,}000 + 20{,}000(1.65) - \left[20{,}000(.05) - 20{,}000(.12)(1.04) \right] \right]$$

$$= 5{,}504.$$

Since the net cash flow from the hedge is negative:

$$20{,}000(.05) - 20{,}000(.12)(1.04) = -1{,}496$$

the company would have been better off not hedging given where the spot price finished.

Question 7.

Answer is D

The accumulated cost of the hedge is (84.30-74.80)exp(.06) = 10.09.
Let x be the market price.
If $x < 0.12$ the put is in the money and the payoff is $10{,}000(0.12 - x) = 1{,}200 - 10{,}000x$.
The sale of the jalapenos has a payoff of $10{,}000x - 1{,}000$ for a profit of $1{,}200 - 10{,}000x + 10{,}000x - 1{,}000 - 10.09 = 190$.
From 0.12 to 0.14 neither option has a payoff and the profit is $10{,}000x - 1{,}000 - 10.09 = 10{,}000x - 1{,}010$.
If $x > 0.14$ the call is in the money and the payoff is $-10{,}000(x - 0.14) = 1{,}400 - 10{,}000x$.
The profit is $1{,}400 - 10{,}000x + 10{,}000x - 1{,}000 - 10.09 = 390$.
The range is 190 to 390. **(Pages 33-41)**

Question 8.

Answer is D

This is based on Exercise 3.18 on **Page 89**. To see that D does not produce the desired outcome, begin with the case where the stock price is S and is below 90. The payoff is $S + 0 + (110 - S) - 2(100 - S) = 2S - 90$ which is not constant and so cannot produce the given diagram. On the other hand, for example, answer E has a payoff of $S + (90 - S) + 0 - 2(0) = 90$. The cost is $100 + 0.24 + 2.17 - 2(6.80) = 88.81$. With interest it is 93.36. The profit is $90 - 93.36 = -3.36$ which matches the diagram.

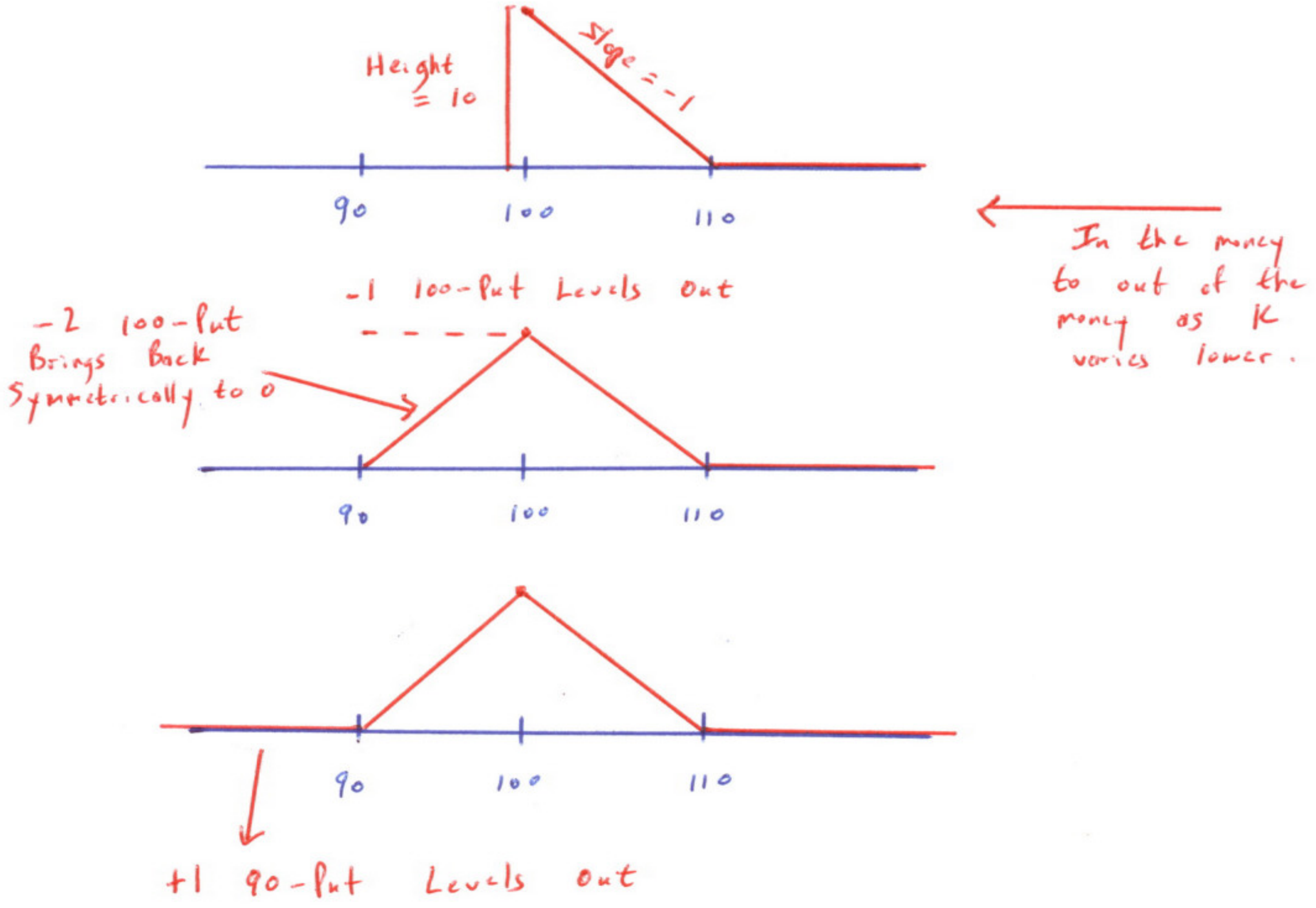
Note:

When analyzing portfolios of puts or calls only, it is best to move from "left to right" (i.e. in the money to out of the money) for calls and "right to left" (i.e. in the money to out of the money) for puts with respect to put strikes and the x-axis position/value of the underlying.

Let us illustrate the point for Question 8.

A.

Put Strike	# Puts
90	1
100	-2
110	1



Shift of Cash Flows is Needed to Produce the Profit Diagram.

Shift = Net Cost of Position Accounting for Interest

$$= [-.24 + 2(1.93) - 6.81]\, e^{.05} < 0$$

so cash flow chart shifts down.

Shift = -3.35 therefore "peak" $= 10 - 3.35$
$= 6.65$.

[To "solve" this problem, first line approach is to find obviously wrong answer.]

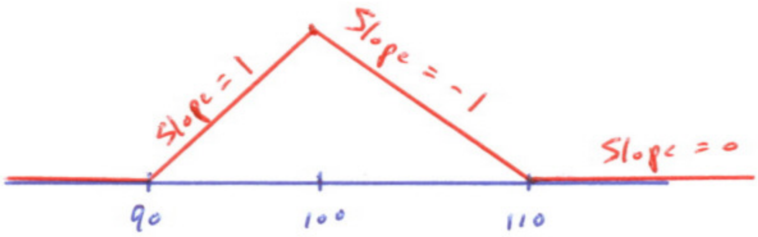
Obviously, the expression for the profit diagram here is:

$$(90 - S_T)_+ - 2(100 - S_T)_+ + (110 - S_T)_+ + \pi$$

where π is net cost with interest ("shift").

B.

Call Strike	# Calls
90	1
100	-2
110	1



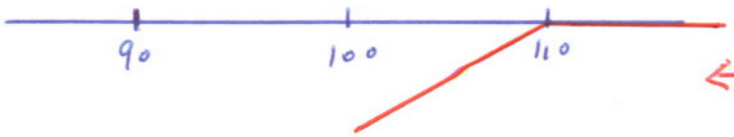
C.

Call strike	# Calls
100	-1
110	1

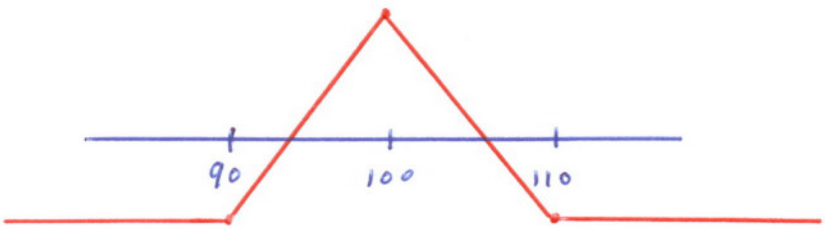
Net Cost is < 0 since $c(100) > c(110)$.

Put Strike	# Puts
90	1
100	-1

Net Cost is < 0 since $P(100) > P(90)$



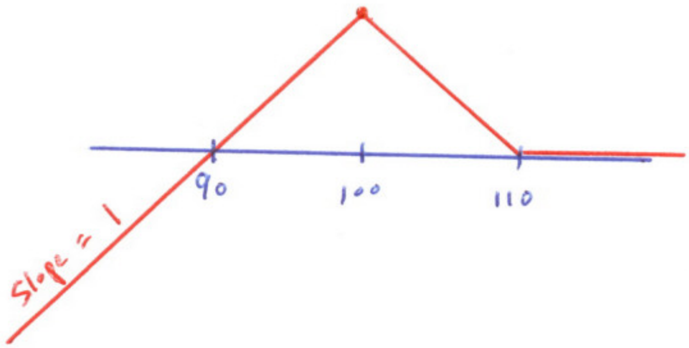
Combined profit position must look like:



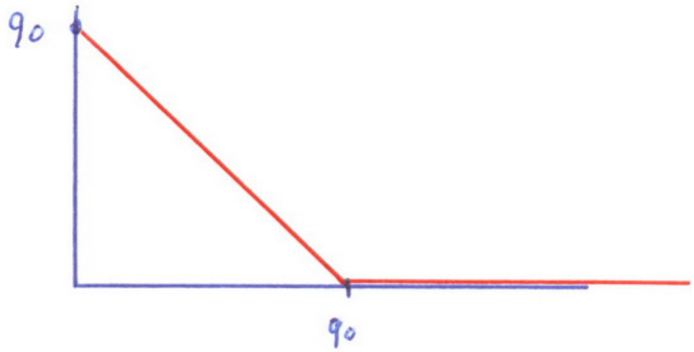
D.

Put Strike	# Puts
100	-2
110	1

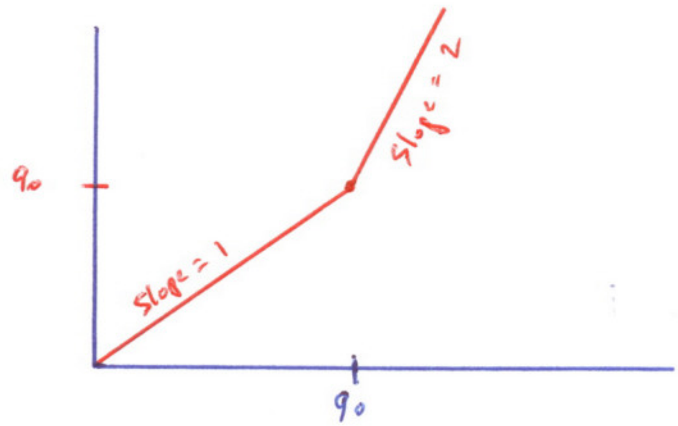
Put Cash Flows:



We are missing the cash flow from a 90-Put to get the correct shape. This cash flow is:



The cash flow from one share of stock and one 90-call is:



[Short Answer: we are short 2 100-puts and long one 110-put. To get the correct cash flows we need the cash flow from long one 90-put. The cash flow from long one share of stock and long one 90-put IS NOT the same as the cash flow from a 90-put.]

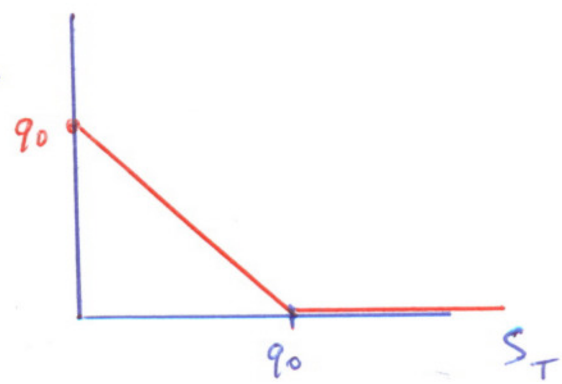
which is not correct. One can also reason using put-call parity. Indeed,

$$(S_T - 90)_+ + 90 = (90 - S_T)_+ + S_T$$

$$\Rightarrow \quad (90 - S_T)_+ = 90 - S_T + (S_T - 90)_+$$

$\therefore$ the cash flow from the 90-put that is needed can also be obtained from buying a 90-call, shorting one share of stock and putting the PV of 90 in the bank (i.e. buy a bond maturing for 90).

$90 - S_T + (S_T - 90)_+$



E.

Call Strike	# Calls
100	-2
110	1

We know that to get the butterfly spread cash flows we need to add the cash flows from a 90-call.

i.e. we need the cash flows from a 90-call.

$$\underbrace{(90 - S_T)_+ + S_T = (S_T - 90)_+ + 90}$$

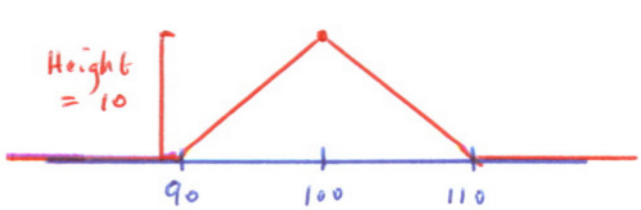
Cash flow from buying one share of stock and one 90-put is the same as the cash flow from buying a 90-call **AND** a bond maturing for 90.

Cost of Position with Interest $= [\,100 + .24 + 2.17 - 2(6.80)]\, e^{.05}$

$= 93.36$

$$\text{Profit} = \underbrace{(S_T - 90)_+ + 90} + (S_T - 110)_+ - 2(S_T - 100)_+ - 93.36$$

$$= \underset{\text{Cash Flow}}{\text{Butterfly Spread}} - 3.36$$



[Note: The extra cash flow from the bond gets "washed out" in the previous adjustment when we compute profit.]